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The three-dimensional structure of the wake of a circular cylinder

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A critical scrutiny of the nature of the three-dimensional characteristics of the vortex wake of a circular cylinder serves to suggest lines for further investigation and furnishes some ideas on the nature of the growth and development of these non-uniformities. It is suggested that the basic occurrence in the growth of three-dimensionality is the continuation of vortex lines, oriented more or less parallel to the body, into the direction of the free stream. The causes of this vary, as do the details of the development with the particular situation considered.

Experiments were performed in a wind tunnel at Reynolds numbers based on cylinder diameter of 85, 235 and 2×10^4, at which stable, transitional and turbulent vortices were investigated.

1. Introduction

The three-dimensional character of the wake of a circular cylinder has been investigated at three Reynolds numbers: at low Reynolds numbers where the wake is stable, at Reynolds numbers in what Roshko (1953) describes as the transition range of vortices, where laminar- and turbulent-flow vortices are present together behind the cylinder at different positions along its length, and thirdly, at a high Reynolds number where the vortices are all turbulent. There are certainly other ranges of Reynolds number where three-dimensional effects arise in a different way. There are, for instance, two other transition ranges; that discovered by Tritton (1959) at Reynolds numbers of about 90, where the change of mode of oscillation is not a two-dimensional transition, and the transition investigated by Humphreys (1960) at the critical Reynolds number, where re-attachment of the boundary layer on the cylinder is again not two-dimensional.

This paper is presented as a first investigation of the cylinder wake by the technique of using an array of hot-wire anemometers distributed along the cylinder length. The descriptions which follow show that the method is certainly a powerful means of investigation of the problem. It is complementary to other methods, such as the measurement of the correlation coefficients between the fluctuations at two points separated along the length of the cylinder, and the methods of flow visualization, which are unsurpassed as a means of detecting the positions of the vortex lines. It seems that a full investigation of the problem would most profitably be performed by a simultaneous employment of all three techniques. For reasons of ease of flow visualization and the desirability of a perfectly uniform stream, it seems that an experiment of the type in which one tows the cylinder through water is the best.

The present experiments were made in a wind tunnel and the main conclusions are that at Reynolds numbers below that of Tritton's (1959) transition at a Reynolds number of about 90 the wake is intrinsically stable and would exhibit a stable two-dimensional character if the flow and model arrangement were two-dimensional. A theory is presented of the three-dimensional characteristics observed due to the non-uniformity of the stream. At these Reynolds numbers, the flapping of the wake is the predominant three-dimensional feature.

At a Reynolds number of 235 the wake exhibits spontaneous three-dimensional characteristics due, it is proposed, to the simultaneous existence of laminar and turbulent vortices. Here, in addition to the flapping of the wake, the difference in strength of the vortices is important in the growth and development of the three-dimensional structure.

At high Reynolds number, where the vortices are all turbulent, variations along the cylinder are less spectacular than at lower Reynolds numbers. The flow has a random character superimposed on a more gentle variation of the low-Reynolds-number type. The mechanics of the growth of three-dimensional effects is expected to be the same for turbulent vortices as for the mixture of laminar and turbulent ones, but there the differences in vortex strength are generally less pronounced and the irregular oscillations generally of smaller amplitude.

The discussion of three-dimensional effects is introduced by a review of existing literature on the subject because such a review has not appeared before and it serves to complete the purpose of this paper, which is to prepare the way for further investigation.

2. Review of existing information

Over the past sixteen years a substantial body of information has been accumulated concerning the three-dimensional characteristics of the wake of a circular cylinder. Two-dimensional flow is only found to exist at the lowest Reynolds numbers, at which a periodic wake is formed. As the Reynolds number of the flow is increased, three-dimensionality enters first in a regular form and later in forms which appear to be random in nature. An important transition Reynolds number ($R \sim 90$) owes its discovery to Tritton (1959). This Reynolds number marks the division between vortex production as a result of wake instability and a vortex production in which the body plays a more significant role. At Reynolds numbers much below 90, Kovasznay (1949) at $R = 50$ and later Phillips (1956) found that the vortex lines were straight and parallel to the cylinder axis for at least 30 diameters. While Tritton is in agreement with this finding at the lower Reynolds numbers, he found a tilted vortex configuration in which the vortex lines were sometimes inclined to the cylinder axis at angles up to 30° at low Reynolds number. Whether the vortices are parallel to the cylinder or inclined to it at low Reynolds numbers is the point on which there

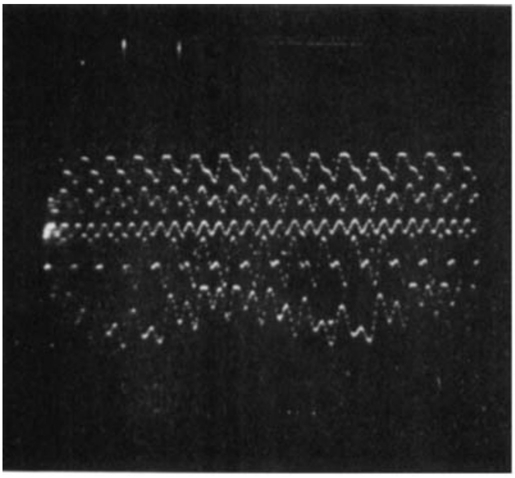

FIGURE 2

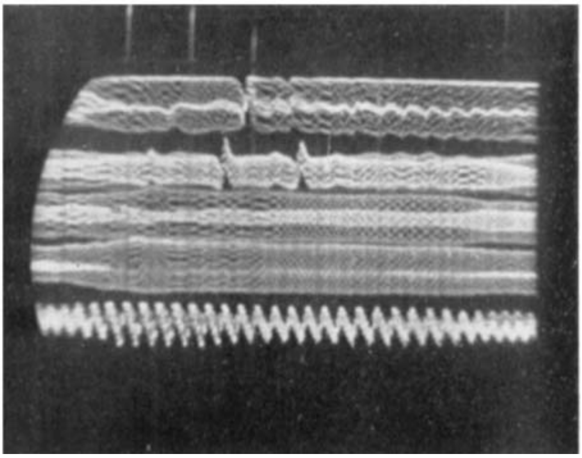

FIGURE 4

FIGURE 2. Cathode-ray oscillogram from five hot-wires placed 4·2 diameters downstream of the cylinder axis at R = 85. Traces are in order as in figure 1, with number 2 at the bottom. Total length of time base = 47 msec. Velocity increases downwards.

FIGURE 4. As figure 2 but with total length of time base = 710 msec.

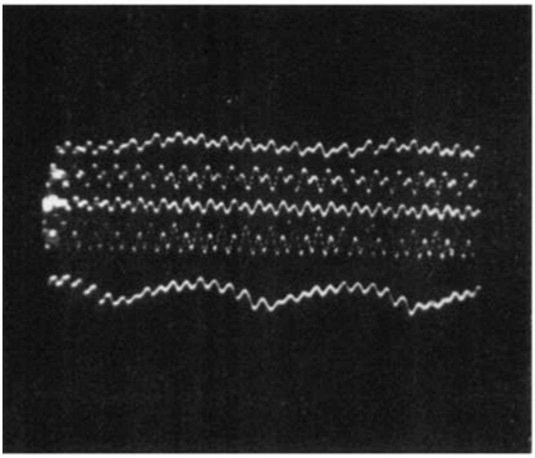

FIGURE 5

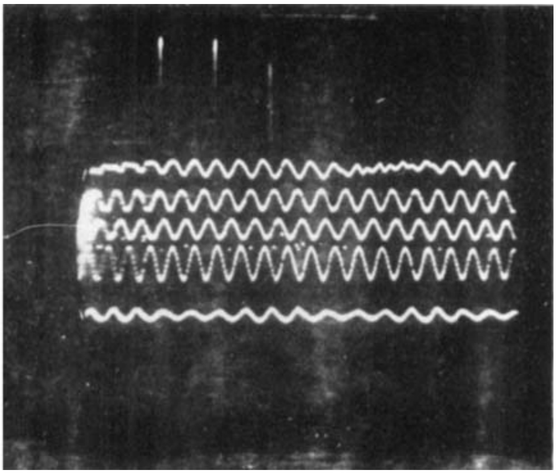

FIGURE 6

FIGURE 5. Cathode-ray oscillogram from five hot-wires placed 17·2 diameters downstream of the cylinder axis at R = 85. Traces in order as in figure 1 with number 2 at the bottom. Total length of time base = 55 msec. Velocity increases downwards.

FIGURE 6. As figure 5, but with hot-wires outside the wake.

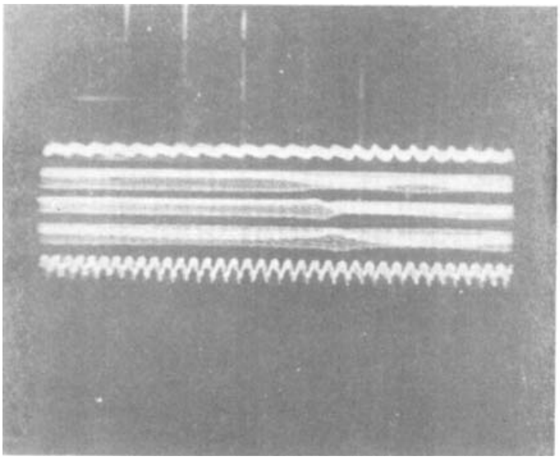

FIGURE 7

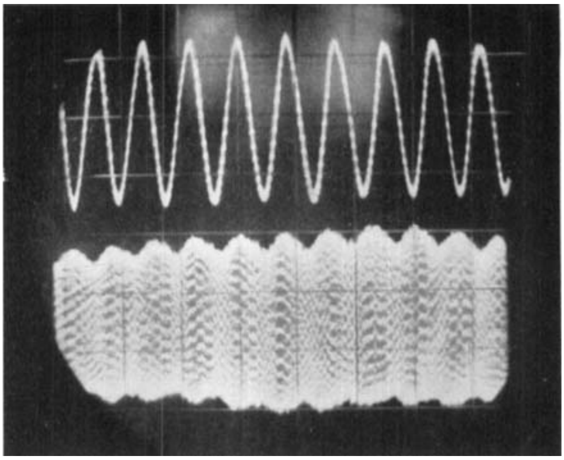

FIGURE 11

FIGURE 7. As figure 5, but with total length of time base = 710 msec.

FIGURE 11. Cathode-ray oscillogram from a single hot-wire placed 26 diameters downstream of the cylinder axis at $R = 113$. Top trace shows fundamental shedding frequency of 6 kcyc./sec. Lower trace obtained with a much longer time base shows the modulation at 200 cyc./sec. Traces obtained at different times by double exposure.

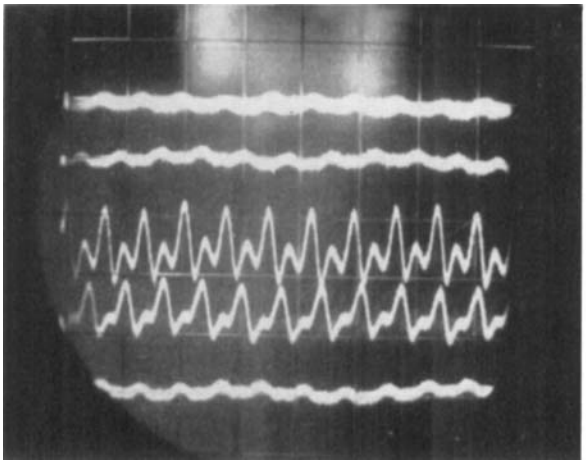

FIGURE 12

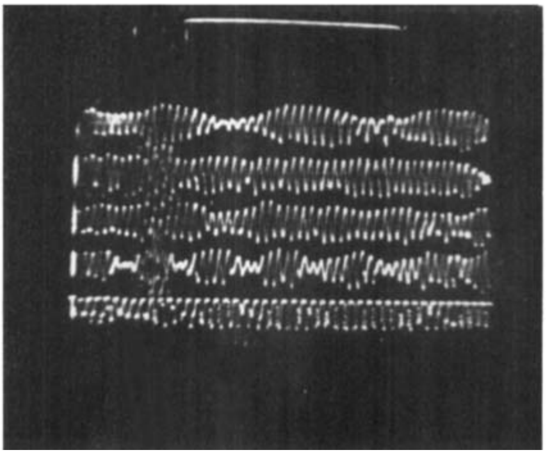

FIGURE 15

FIGURE 12. As figure 11, but with hot-wire 450 diameters downstream. Top two traces and bottom trace; hot-wire outside the wake on opposite sides (frequency = 200 cyc./sec.). Middle two traces inside the wake shows 400 cyc./sec. signal. Single hot-wire multiple exposure.

FIGURE 15. Cathode-ray oscillogram from five hot-wires placed 4·25 diameters downstream of the cylinder axis at $R = 235$. Traces are in order as in figure 1, with number 2 at the bottom. Total length of time base = 315 msec.

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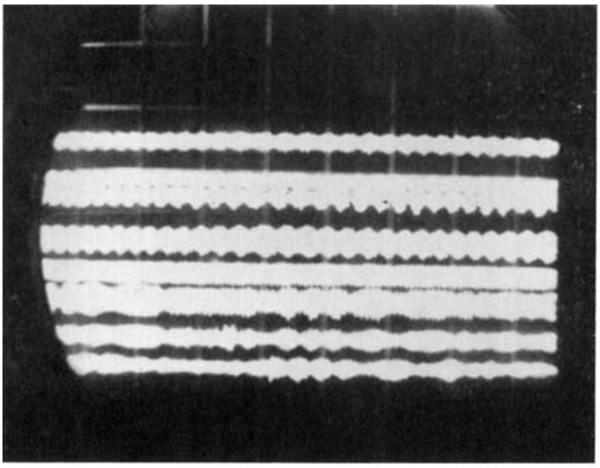


FIGURE 16

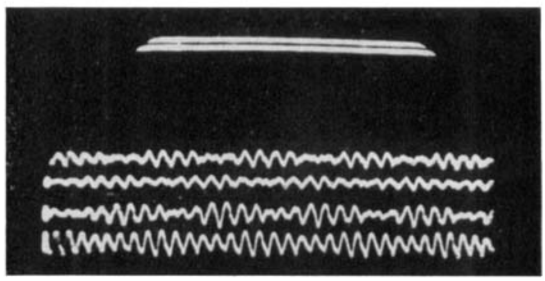
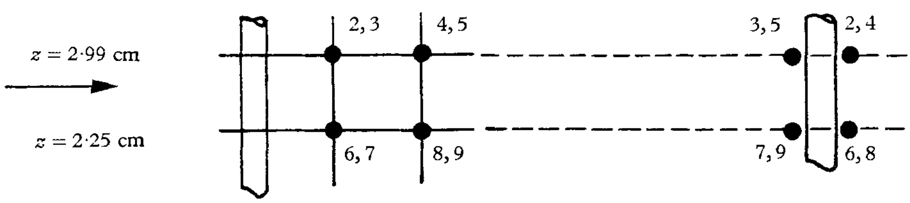


FIGURE 17



FIGURES 16–17. Cathode-ray oscillograms obtained at $R = 235$ with the experimental arrangement of figure 1, with eight hot-wires grouped as shown by the dots below and marked 2 to 9. Only four hot-wires at the same value of z were used in figure 17.

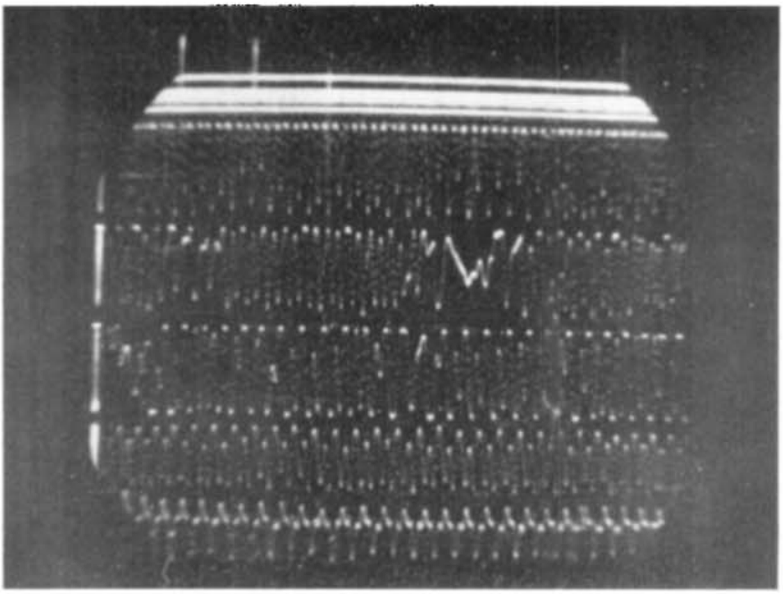


FIGURE 19. Cathode-ray oscillogram from five hot-wires placed 1·75 diameters downstream of the cylinder axis at $R = 235$. Traces in order as in figure 1, with number 2 at the bottom. Total length of time base = 236 msec.

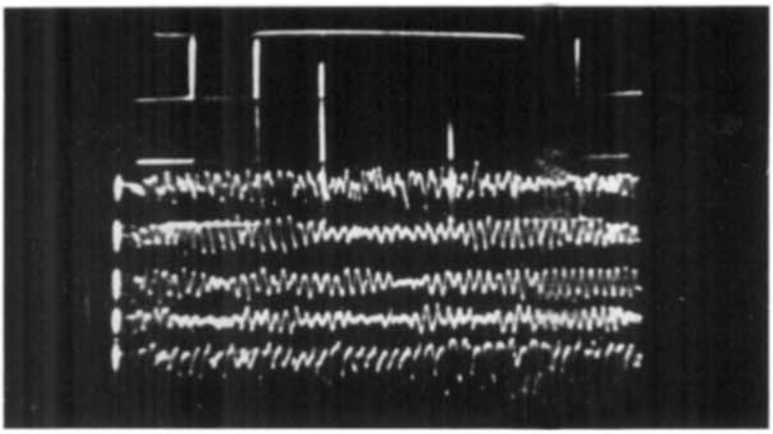


FIGURE 21. As figure 15 but with the hot-wire 12·3 diameters downstream of the cylinder axis.

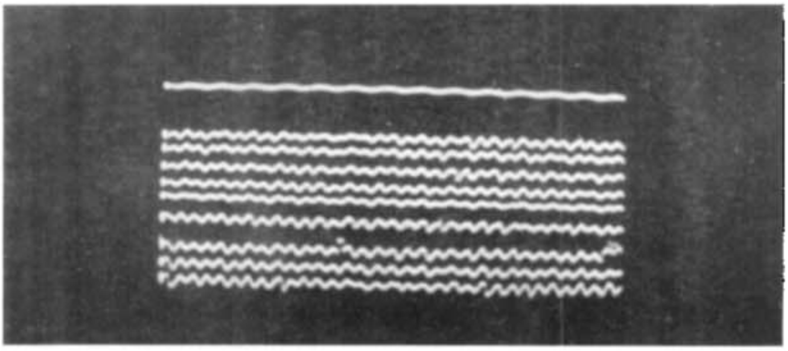


FIGURE 22. Cathode-ray oscillogram from hot-wires 3 diameters downstream of the cylinder axis and separated along the axial direction by one diameter at $R = 2 \times 10^4$. Cylinder spanning the working section.

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seems to be most conflict. Hama (1957) finds the parallel configuration at R = 117; Taneda (1952) finds the parallel configuration at $R \leqslant 60$ but vortices inclined at about 10° at $R = 75$. It is perhaps significant that parallel configurations are found at $R > 50$ only in experiments where the cylinder is towed through water. Phillips (1956), in his towing experiment, discovered that if the water was allowed to settle so as to be free from disturbances, then two-dimensional flow could be observed up to $R = 100$; when the cylinder was towed through disturbed water two-dimensional flow was only observed up to $R = 80$. This suggests that the straight and parallel vortex configuration is only observed when the fluid is free from disturbances. There are, however, two other possible explanations: one is the effect of end conditions. In the towing experiments there is no side-wall boundary layer. Fage (1934) in a water-channel experiment discovered that three-dimensional effects were present in embryo within the attached vortices at a Reynolds number as low as 35. This effect, of spanwise flow within the vortices, seems to have been a result of the end-wall boundary layer; that is, the free stream is not two-dimensional right to the ends of the cylinder. On the other hand, two-dimensionality can be forced on the flow as a result of cylinder oscillation. Berger (1964) has shown in a wind tunnel that the vortex lines were inclined at about 19° to the cylinder. When the cylinder was forced to oscillate the vortex lines became parallel to the cylinder axis. In experiments where the cylinder is towed through water it is more likely to vibrate than when in air, the oscillating side force being larger in the ratio of the fluid densities.

At Reynolds numbers greater than Tritton's transition value at about R = 90, all workers find three-dimensional effects, with the exception of Hama's experiments. Phillips's quiet water experiments may elevate the transition Reynolds number: the same may be said of Hama's experiments. Roshko (1953) and Phillips (1956) investigated the wake at two points separated in the spanwise direction. They found a phase shift between the oscillation at the two points. Roshko reports a wavelength in the direction of the cylinder axis of 18 diameters at $R = 80$, Phillips reports one of 15 to 20 diameters for $100 < R <$ 150. From these investigations we cannot determine whether the vortex lines are wavy or inclined. Other authors have used flow-visualization techniques which resolve this point in their experiments. In Hama's work the vortex lines are very slightly inclined (at $< 10°$) but are wavy with a wavelength of about 10 diameters at $R = 190$. Taneda's vortex lines are almost free of waviness but are inclined at about 10° at $R = 75$. Berger's vortex lines are almost free of waviness and are inclined at about 19° but exhibit kinks, as do those of Tritton. At Reynolds numbers above the transition value he discovered, Tritton remarks that kinks develop spontaneously as the vortices travel downstream. Abernathy (1964) has made some of his photographs of hydrogen-bubble flow visualization available to the author. These, which are said to be typical, were taken at Reynolds numbers between 100 and 140 and show inclined vortices. One picture shows vortices inclined at 8°; two others show vortices parallel to the cylinder over part of its length with an abrupt change to an inclination of 15° in one case and 30° in the other.

At Reynolds numbers above 150, where we enter the irregular range of vortices in which vortices composed of turbulent fluid begin to appear, there is less disagreement in the results of the available experiments. This is perhaps partly because the chaotic nature of the three-dimensional structure allows less precise description of the phenomena. There are few flow-visualization results in this range. Hama's highest Reynolds-number photograph is at $R = 313$, in which we see the 10-diameter wavelength still persisting but accompanied by waves of much shorter length. Between Reynolds numbers of 3×10^3 and 10^5 Mattingley (1962) finds a periodicity in the direction of the axis on the cylinder surface, the wavelength is a few diameters but no quantitative description is given. The spanwise structure became chaotic as the critical Reynolds number was approached.

Most authors describe the spanwise structure of the turbulent vortices in terms of a correlation length, this being the length over which velocity fluctuations in the wake may be described as perfectly correlated. Points separated by more than the correlation length are regarded as having zero correlation. Thus Roshko (1953) gives the rough values for correlation length as 10 diameters at $R = 220$ and 3 diameters for $R = 500$. Phillips finds a correlation length of about 3 diameters at $R = 5 \times 10^3$. Prendergast (1958) and el Baroudi (1960) find that between Reynolds numbers of 10^4 and 10^5 the correlation length is about 4 diameters. Using threads attached to the cylinder to visualize the flow, Humphreys (1960) found a stationary pattern of wavelength 1·4–1·7 diameters at the critical Reynolds number. Without the threads in place there was some evidence that this cellular pattern engaged in a somewhat random motion back and forth along the cylinder.

3. Experimental procedure

The wind tunnel used in these experiments was the 20 in. × 20 in. working section, low-turbulence wind tunnel previously described (Gerrard 1965). Very low speeds are required for the investigation of flow past a circular cylinder at low Reynolds numbers in air, if the scale of the experiment is not going to be very small. It was also necessary that the vortex-shedding frequency be as low as possible (less than 1000 cyc./sec) because of the form of the electronic sampling arrangement designed to take the signal from ten hot-wires sequentially.

Initially, experiments were made with cylinders spanning the working section. It soon became obvious that at low wind speeds the cylinder wake was a good detector of the variation of the free-stream velocity along the cylinder length. One can in fact determine the velocity distribution across the wind tunnel by measuring the frequency of vortex shedding at points along the length of a cylinder spanning the tunnel. It was found that at low speeds of about 100 cm/sec the velocity distribution was far from uniform and changed with the wind speed. In view of the shortness of time available, an investigation into the cause of this was not pursued, but instead a speed was chosen at which $7\frac{1}{2}$ in. of the 20 in. working section had a uniform velocity distribution. The cylinders were mounted between end plates in this uniform velocity region. The end plates were RAF 30 aerofoil sections of 8 in. chord and $6\frac{1}{2}$ in. span. The cylinders spanned the gap just upstream of the maximum thickness of the aerofoils (see figure 1). Some non-uniformity of flow was expected due to the varying cross-section of the gap, but the results show that the velocity was uniform over a large part of the cylinder length. End effects, due to the boundary layer on the aerofoils, were minimized with this arrangement, since the aerofoils were inclined so that the area of the gap decreased in the stream direction to just past the cylinder position. Flat end-plates with flaps could have produced a more uniform flow. One must, however, consider this improvement in the light of the time required to set the flaps correctly. As it transpired, the inclination of the flow to the tunnel axis near the ends of the cylinder produced interesting and illuminating effects.

The cylinders used in the main experiments at low Reynolds number were hypodermic tubing of diameter 0·086 and 0·235 cm. The high-Reynolds-number experiment used a 1 in. diameter brass cylinder which spanned the whole working section.

Up to ten hot-wire anemometers have been used simultaneously to investigate the three-dimensional structure of the wakes. The hot-wires derived their constant heating current from a battery through large resistors. The signals from the hot-wires were amplified by simple a.c. transistor amplifiers. This was a perfectly adequate arrangement since we are not interested in the absolute amplitude of the signals and the highest frequency of interest was only about one-third of the hot-wire cut-off frequency. Each amplifier output was connected into one of ten different cathode circuits of a counting tube operated at 100 kcyc./sec. The signal level and the d.c. level at each cathode could be varied. In this way each signal appeared at the anode of the counting tube at a frequency of 10 kcyc./sec. The output from the anode was displayed on a cathode-ray oscilloscope. Thus a single-trace cathode-ray oscilloscope was converted into a ten-trace tube, from which the relative phase of the signals could be measured.

The hot-wires were etched Wollaston wire of core diameter 0·0001 in. Each wire was supported on a separate holder. The holders were held in metal blocks carried on a slide which was set parallel to the cylinder axis. The slide was encased in an aerofoil housing and its position could be varied along the free-stream direction and perpendicular to this and the cylinder axis. The holders were positioned so that the wires lay as nearly as possible on a line parallel to the cylinder axis. The streamwise co-ordinates of the wires were measured with a travelling telescope and corrections made to the results for the small departures from equality of this co-ordinate. The differences in position in the direction perpendicular to both the free stream and the cylinder axis were small, and in this condition, of no consequence. The separation of the wires in the direction of the cylinder axis was also measured with the travelling telescope.

With the cylinders between the aerofoil end-plates the number of hot-wires was restricted to 5 in order not to produce too much blockage of the flow. Minimum intereference to the wake was obtained by mounting the hot-wire holders at about 30° to the plane of the wake. The aerofoil housing at the end of the holders remote from the wires was at zero incidence to the free stream.

In all the measurements at Reynolds numbers of 85 and 235 the positions of the hot-wires along the cylinder axis was as shown in figure 1. In this figure the wires are given numbers to which reference is made in what follows. It is noteworthy that wire 2 is much nearer to the aerofoil boundary than is wire 6.

The fact that the frequency of vortex shedding from a circular cylinder is appropriate to the free-stream speed at this section of the cylinder has already been mentioned. In the presence of spatial non-uniformity of the approaching stream a two-dimensional wake structure cannot exist because more vortices per unit time are cast off one section of the cylinder than another. This three-dimensional structure will persist even for very small degrees of spatial non-uniformity. Turbulent fluctuations of the free stream will likewise produce three-dimensional effects. Turbulent fluctuations in the longitudinal or transverse direction would need to be several per cent of the free-stream speed in order to produce three-dimensional effects as big as those observed here, and so the effects of free-stream turbulence may be ignored.

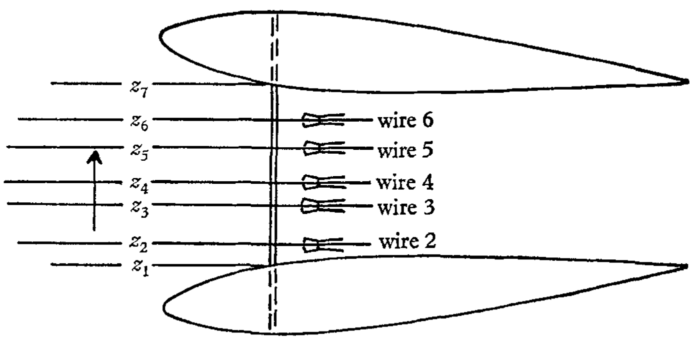

FIGURE 1. Experimental arrangement and hot-wire positions. $z_1 = 0$ cm, $z_2 = 0{\cdot}595$ cm, $z_3 = 2{\cdot}05$ cm, $z_4 = 3{\cdot}09$ cm, $z_5 = 4{\cdot}79$ cm, $z_6 = 6{\cdot}22$ cm, $z_7 = 7{\cdot}5$ cm.

4. Presentation and discussion of experimental results

The main experiments were made at Reynolds numbers, based on cylinder diameter, of 85, 235 and 2×10^4. At the two lower Reynolds numbers the free-stream flow was identical and the cylinders were mounted between the aerofoil end-plates described above. The high-Reynolds-number experiment was made with the 1 in. diameter cylinder spanning the wind-tunnel working section. The Reynolds number of 85 is in what we choose to call the intrinsically stable régime of flow, in which, with a two-dimensional experimental arrangement, the wake is expected to be two-dimensional also. The Reynolds number of 235 lies in the unstable régime of wake flow and three-dimensional effects are inevitable there. At Reynolds numbers greater than about 400 the wake vortices are composed of turbulent flow. The results at a Reynolds number of 2×10^4 are thought to be typical in quality of the turbulent range.

4.1. *Reynolds number* = 85

Despite the proximity of this Reynolds number to the transition value discovered by Tritton (1959), the effects associated with this transition appear to be absent. Measurements were made close behind the cylinder and some distance downstream with hot-wires inside and outside the wake. Figure 2 (plate 1) shows a cathode-ray oscillogram obtained with the five hot-wires 4·2 diameters downstream of the centre of the cylinder of diameter 0·086 cm. In all cases the traces will be referred to by numbers 2 to 6 from bottom to top, as in figure 1. The relative proportions of fundamental and first harmonic frequency show how well the wires were placed in line. Trace 4 exhibits pure first-harmonic signal owing to the wire being on the central plane of the wake: the others show varying amounts of fundamental and first harmonic. It is not known which side of the central plane the wires lay: the estimated maximum distance of the wires from this plane was less than 0·25 mm.

The oscillograms were measured by projection onto graph paper. Figure 3, obtained from figure 2, shows the variation of the fundamental period with time. The period units are arbitrary but the same for each trace. Five identical curves are drawn: their curvature is due to non-linearity of the time base, which was rectified later. The average periods are shown on figure 3. Those of traces 3 to 6 appear identical as far as our accuracy of measurement will show. Points with different symbols on traces 2 and 3 indicate the repeatibility of measurement of the oscillogram; similarly, in figure 8. The modulation of trace 2 in figure 2 is reflected in the variation of period with time in figure 3. The mean period on trace 2 lies above that for the other traces and the period oscillates with the same periodicity as the modulation in figure 2. Examination of figure 2 reveals that in one period of the modulation on trace 2 there are 6 fundamental periods on trace 2 and 7 periods on all the other traces. The ratio of the average periods is 1·160 which is close to the ratio 7/6 = 1·167. Evidently the modulation is associated with the difference in mean period, which in turn is directly related to the difference in free-stream conditions.

The modulation waveform is included in figure 3. Comparison shows that maximum and minimum of modulation and of the period occur together but in antiphase. Velocity increases correspond to negative excursions of the traces in figure 2 so that the maxima of the modulation correspond to minimum speed. One also notices that these positions of minimum speed correspond to relatively more first-harmonic signal than do the maximum speed regions. Minimum speed and maximum first-harmonic content are found at the centre of the wake. (The flattening of the tops of the waves on traces 2 and 6 are due to overloading of the amplifiers.) We notice that when the centre of the wake is near the wire 2 the period is closest to being equal to that of the other traces.

Figure 4 (plate 1), taken with a similar configuration to figure 2, shows the result of lengthening the time base, The amplitude and waveform on traces 3, 4 and 5 do not in fact remain constant, but show variations. Figure 4 extends over about 200 fundamental periods. The amplitude and waveform over the central region of the cylinders remain constant for times of the order of 100 periods, and show changes which take about 10 periods to be completed. These transitions occur at about the same time on all traces. In connexion with figures 2 and 3, we saw that the period of the modulation on trace 2 was such that there was one difference between the integral number of fundamental periods in the modulation interval on trace 2 and the same interval on other traces. The modulation with a period of the order of 100 fundamental periods seems to fit in with a difference in period of about 1 % on traces 3, 4 and 5. In the whole of the work at low Reynolds number it was found that whenever the amplitude of the

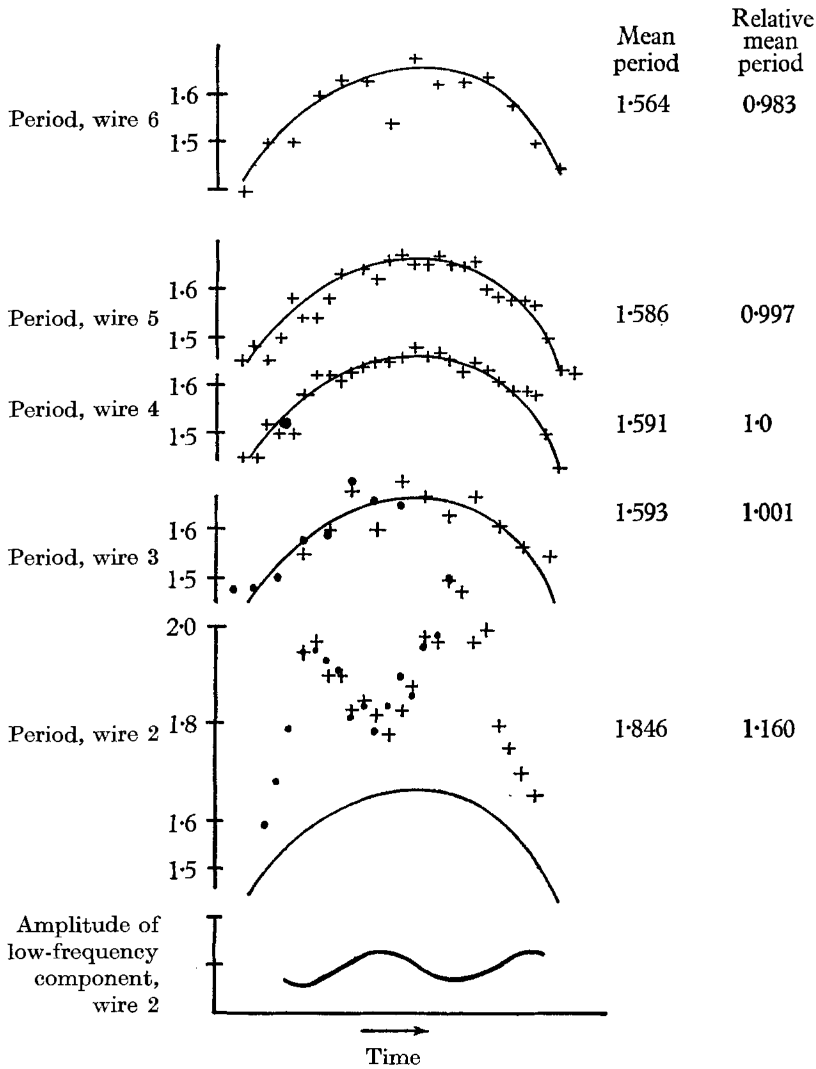

FIGURE 3. Periodic time as a function of time, from figure 2. (The curvature of the five identical curves was due to non-linearity of the time base, also in figures 8 and 9.)

trace varied, the waveform varied simultaneously. With the long time base of figure 4, we see the envelopes of the waveforms: the band down the centre of the traces is at first harmonic frequency. The simultaneous change of amplitude and waveform can be clearly seen. We infer from this that the wake flaps like a flag and that the modulation observed is associated with the transverse motion of the wake centre line: we thus rule out the other possibilities that the modulation is produced by fluctuations in vortex strength or the superposition of a low-frequency velocity field, the wake geometry remaining fixed.

We now see that the modulation on figure 2 is simply the highest frequency modulation; one of an order of magnitude lower frequency is evident on trace 2

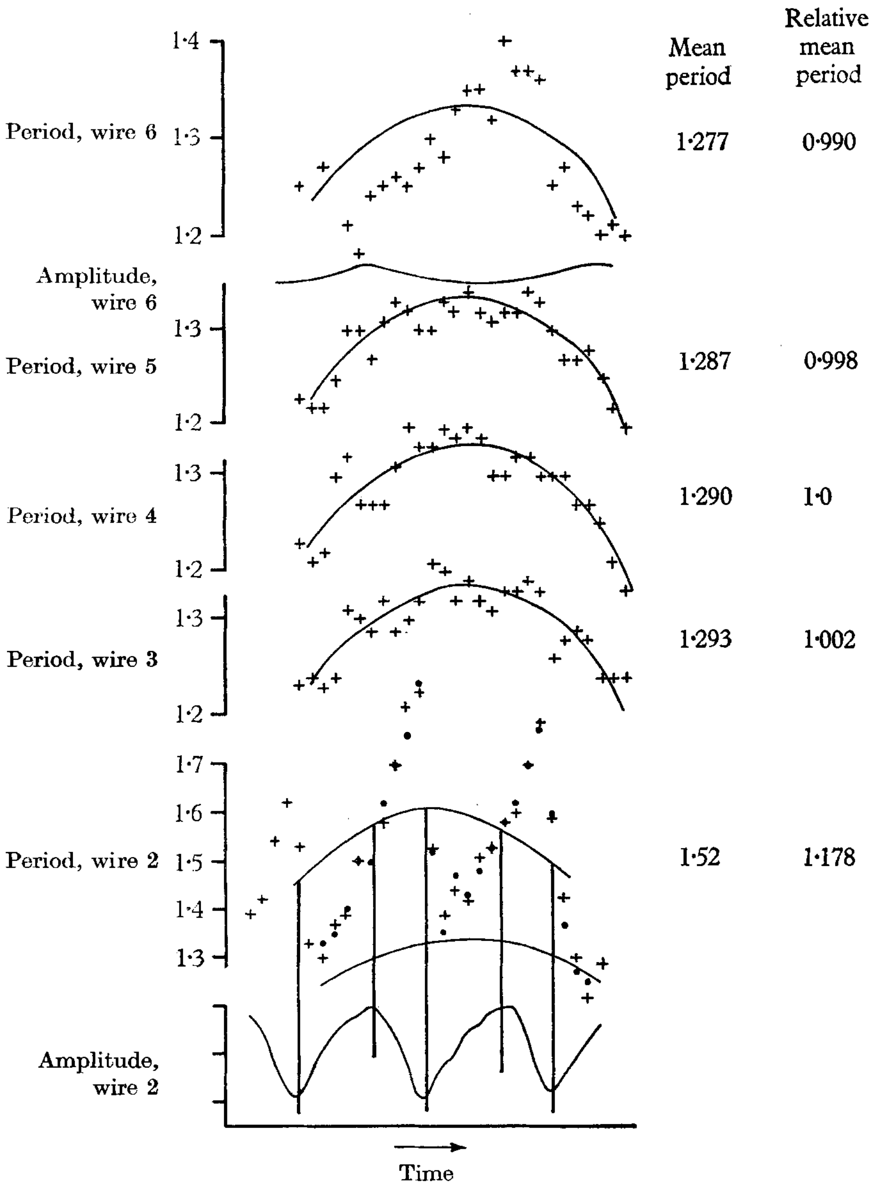

FIGURE 8. Periodic time as a function of time, from figure 5.

in figure 4. Though no modulation was present on trace 6 of figure 2, we see from figure 4 that modulation is intermittently present with about the same period as that observed on trace 2. The two jumps on trace 5 and one on trace 6 are not understood. They bear a slight resemblance to disturbances observed by Tritton (1959) but it is not impossible that they are due to electrical interference.

Figures 5, 6 (plate 1) and 7 (plate 2), obtained with the same relative hot-wire positions but at 17·2 diameters downstream, show how the wake has developed downstream. The modulation in figure 6 takes on a different appearance because the hot-wires were outside the wake. We see here the effect of the wake approaching and receding as a change in amplitude without much change in mean level. The waveform changes from the form $A \sin \omega_1 t + B \sin \omega_2 t$ to one in which the

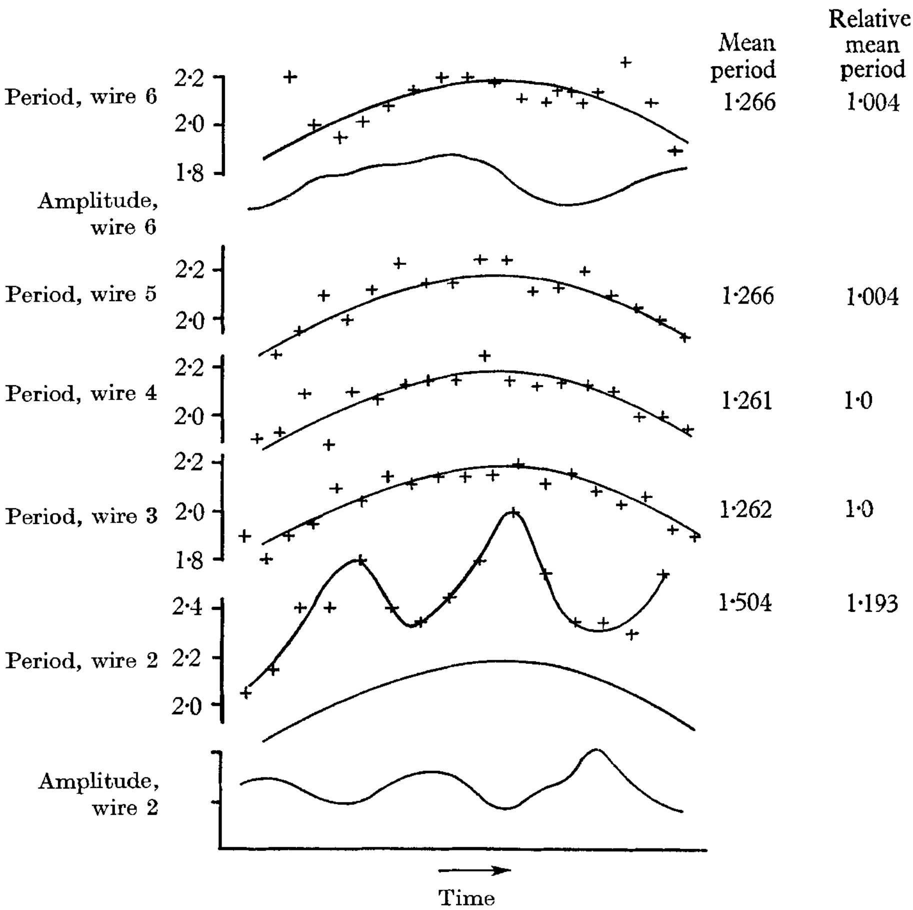

FIGURE 9. Periodic time as a function of time, from figure 6.

predominant component is of the form $A \sin \omega_1 t \sin \omega_2 t$. The modulation of trace 6 has increased so as to be present at all times. The downstream development involving the encroachment of end conditions into the part of the wake with constant mean properties has been demonstrated by Taneda (1952).

The transitions on the three centre traces of figure 7 have developed a little from the upstream position of figure 4. There is still constant amplitude for the order of 100 periods, but the transitions take a little longer to complete and are separated in time by as much as 20 to 30 periods. Figures 8 and 9, constructed from figures 5 and 6, show a variation of the fundamental period with time very similar to that observed close to the cylinder, with the exception of trace 6.

This trace shows oscillations in phase with the modulation waveform in figure 8: the oscillation is about a mean value equal to the mean period of the central traces. Figure 8 refers to hot-wires inside the wake; in figure 9 the wires were outside the wake: otherwise the conditions were identical and we expect no difference between figures 8 and 9. In figure 9, trace 6 is not very regular with respect to either modulation waveform or the variation of period. The amplitude is certainly not a maximum where the period is a minimum. Traces 2 of figures 8 and 9 show a marked regularity but the modulation is maximum when the period is least in figure 9, whereas in figure 8 the maxima and minima of the modulation occur almost exactly half-way between the peaks and troughs of the period variation. We know from figures 4 and 6 that the modulation is not constant on traces 2 and 6. Figure 9 may have caught trace 6 at a transition. Do the two conditions of trace 2 in figures 8 and 9 correspond to two equilibrium states, or is there a gradual phase change between period and modulation? Obviously further investigation is needed.

Vortex-line positions may be inferred from hot-wire measurements provided we use enough hot-wires and their separation in the direction of the cylinder axis is small enough. If the vortices are convected at the same speed at different spanwise stations the oscillogram representing the time history at a given downstream station may be interpreted as a spatial distribution at a given time. The scale of the spatial distribution in the streamwise direction is proportional to the speed of convection of the vortices. If the vortices in the neighbourhood of a particular spanwise station are moving at a different speed from the rest, the spatial representation of these vortices varies with time. An instantaneous picture at one particular time is obtained from the time history by shifting the vortex times by an amount proportional to the velocity of convection and also by an amount proportional to the time, measured from some appropriate origin.

For the determination of vortex positions, oscillograms obtained with the hot-wires outside the wake are superior because when the wires are near the wake central plane it is not known on which side of this plane they lie, and hence one cannot distinguish between positive and negative vortices. In figure 10 the positions of the peaks of the waveforms in figure 6 are plotted with separation of the rows proportional to the actual separation of the wires. The points have been subjected to a small correction for the differences in the hot-wire positions in the streamwise direction. This was done on the assumption that the vortices are convected with 0·86 times the free-stream speed and using the measured frequency of vortex shedding.

It is immediately apparent that the vortex lines are not parallel to the cylinder axis. They could be wavy, but equally they could be straight and inclined. The inclination of the vortex lines corresponding to the lines in figure 10 is 14°. The tangent of this angle is proportional to the vortex speed, which was taken to be 0·86 times the free-stream speed. The lines drawn in figure 10 are all parallel. They pass through, or very close to, all the points in rows 3 to 6. The fact that straight vortex lines pass through all four sets of points seems to be a strong indication that the vortex lines are in fact straight. A striking phase relationship is seen to exist between these lines and the points plotted from trace 2. These points represent a spatial distribution only if the vortices are convected at the same speed as those in the other traces. They are in phase with the vortex lines drawn at the maximum of the amplitude and out of phase at the minimum. The difference in mean period at position 2 can be reconciled with no difference in convection speed if the different frequency at position 2 were due to flow inclination rather than speed difference. This would involve an inclination of the velocity vector of about 30° at station 2 relative to the central stations. This seems to be quite reasonable from a consideration of the model configuration (figure 1).

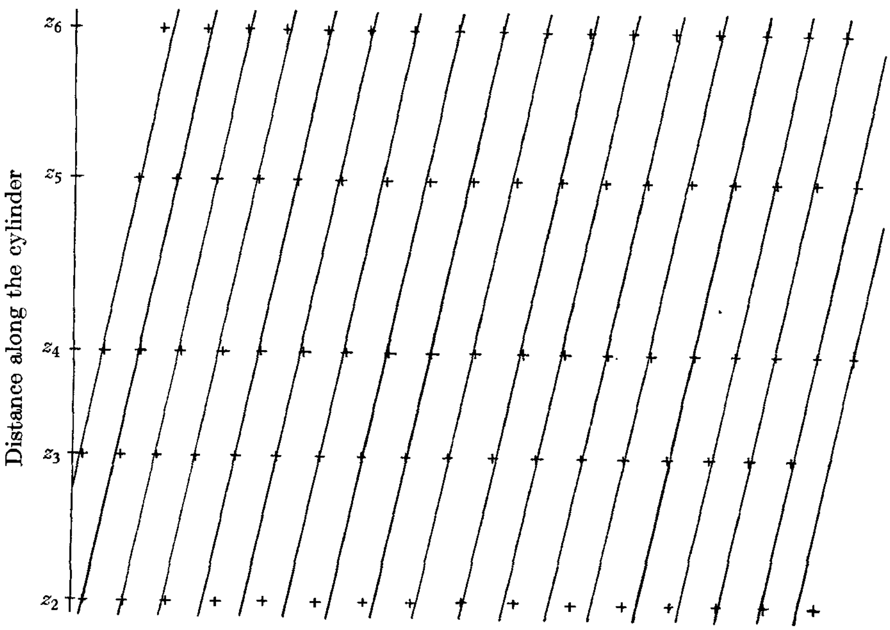

FIGURE 10. Vortex lines from figure 6. The points indicate the times of the peaks of the waveforms in figure 6.

It is interesting to note that if the wires are spaced evenly along the length of the cylinder it is impossible to define the angle of inclination of the vortex lines unambiguously. If, with even spacing of the wires, a certain inclination fits the points, then lines of greater inclination will also fit just as well. This is not the case when the wires are not evenly spaced.

Graphs similar to figure 10 have been constructed from all the other oscillograms. With the wires near the wake central plane there is more ambiguity in the possible vortex-line positions. Always one can draw straight, or almost straight, vortex lines fitting all five sets of points as in figure 10. The inclination of the possible lines varied from about 0° to about 30°, though an inclination of about 14° could be fitted to all the data. The transitions at stations 3 to 5 seen in figures 4 and 7, suggest that the inclination of the vortex lines may be altering with time. This possibility has not been investigated.

4.2. *Downstream development at Reynolds number* = 113

The results presented so far show that the wake flaps from side to side like a flag. In this connexion an interesting observation was made very far downstream in the wake, at a Reynolds number of 113. With a wire of diameter 0·033 cm spanning the wind-tunnel working section, a velocity was found at which the modulation of the signal from a hot-wire in the wake was sinusoidal. Figure 11 (plate 2) shows two traces obtained at different times from one hot-wire. The top trace shows the fundamental frequency of 6 kcyc./sec (the dotted appearance is due to the 200 kcyc./sec heating current of the anemometer used). The lower trace, with much longer time base, shows up the modulation frequency of 200 cyc./sec. As at the lower Reynolds number discussed in § 4.1, a modulation at an order of magnitude smaller frequency was also present. The oscillogram of figure 11 was obtained at 26 diameters downstream from the wire shedding the vortices.

Figure 12 (plate 2) was taken at 450 diameters downstream. Again, several traces are produced from one hot-wire by multiple exposure and vertically shifting the time base. The fundamental frequency gradually disappears with increasing distance downstream and is lost in the background noise at this distance. The modulation frequency, however, persists at about the same amplitude. The top two and the bottom traces of figure 12 were obtained with the hot-wire outside the wake on opposite sides and show the 200 cyc./sec wave. In the centre of the wake we obtained the middle two traces, which clearly show the existence of the first harmonic at 400 cyc./sec. This shows that the wake centre line is sinusoidal. The possibility that a vortex street of frequency 200 cyc./sec has been formed is not ruled out.

4.3. *Discussion of low-Renolds-number results*

An explanation of the results discussed in § 4.1 will be proposed which depends mainly on the finding that amplitude changes are the result of oscillations of the wake medial plane. Amplitude changes are always associated with waveform changes when the hot-wire is near the centre of the wake. In some circumstances the modulation of the waveform has a period such that the number of fundamental periods in one modulation period is one different from that on an adjoining unmodulated trace. Taneda (1952) has shown that vortex lines behind a cylinder with a free end do not continue beyond the end of the cylinder. He postulated the existence of closed vortex loops which by their mutual distortion introduce a three-dimensional effect further along the cylinder as the wake progresses downstream.

The cause of the difference in mean vortex-shedding frequency at different points along the cylinder determines the details of our explanation. The simplest result follows in the case of variations along the cylinder length of the inclination of the flow in the plane containing the free-stream velocity vector and the cylinder axis. In this situation, the effective velocity varies along the cylinder length but if the flow becomes parallel close behind the cylinder, as it does in these experiments, the convection speeds of the vortices are not different. In this case, time history and spatial configuration behind the cylinder are similar. The same effect would be produced if in a uniform stream the diameter of the cylinder varied along its length. Experiments by Abernathy (1964) with a tapered body of circular section show that the shedding frequency is that appropriate to a two-dimensional body of diameter equal to the local diameter.

Consider the case, therefore, where the wavelength in the wake is constant but different at two positions along the length of the cylinder. There will inevitably be a longer wavelength periodic structure, over which the two sets of vortex lines go in and out of phase. Such a situation with a wavelength ratio of 5/4 is shown in figure 13, in which straight full lines represent positive vortex lines and dashed lines negative ones. The difference in frequency is 20 %, which is produced either by a difference in diameter of 20 %, or an inclination of the stream at an angle $\cos^{-1} 0{\cdot}8$. There will therefore be a difference in the strength

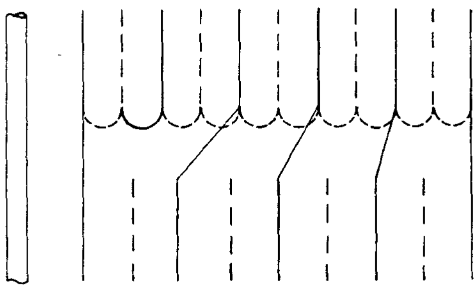

FIGURE 13. Vortex lines when there is a step change in cylinder diameter or a change in flow inclination.

of the vortices, proportional to velocity times diameter, of 20 %; the strongest vortices also being the more plentiful. Because of the difference in strength, the strength of the vortex lines will bend round 20 % to form closed loops, as shown by the semi-circles in figure 13. However, the effect of these loops would seem to be secondary here.

Consider the time at which vortices are shed together at the cylinder. One period later the transition will be crossed by a slightly inclined vortex line. Figure 13 shows development with time if one considers time increasing from right to left. As time progresses the inclination of these vortex lines across the region of transition increases until, if the process is continued in the same way, after one period of the modulation the vortex lines would be one period out of step. At, or before, this stage, the vortex lines cannot join along the inclined path because the earlier vortex has formed before the later one has started to grow. The surplus circulation must therefore loop over and join with the negative vortex of its own set. This looping over will almost certainly not be confined to one short period but will be distributed. Half-way through this modulation period, exactly the same process occurs in antiphase with the vortices of the other sign. The vortex loop causes motion of the medial plane of the wake in opposite directions above and below the loop. Close to the transition the wake centre line will be sinusoidal: the sinuosity is fixed relative to the vortices and is in antiphase above and below the transition. The phenomenon has been considered in terms of an abrupt change in free-stream conditions. The scale of the change will presumably have an effect, since this determines the slope of the vortex lines in the region of transition.

No explanation is offered of why the short period oscillates in phase with the modulation period. No doubt the frequency-determining mechanism will be affected by the periodic structure we have described.

It seems likely that in the results presented in § 4.1, the hot-wire 6 lies on the high-frequency side of the transition region of figure 13 and hot-wire 2 on the low-frequency side. The system of vortex loops will spread into the uniform part of the wake, as Taneda (1952) describes, which explains why the modulation is found to spread in our experiments.

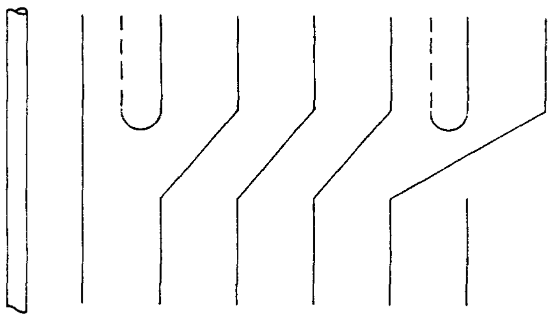

FIGURE 14. Vortex lines when there is a step change in free-stream speed with position along the cylinder. The diagram assumes the pattern formed at the cylinder is convected without change.

There remains the possibility that the difference in frequency at two positions along the cylinder is due to a different free-stream speed. In this case, exactly the same mechanism will occur close behind the cylinder as that described above, and the same conclusions about vortex strength apply, that is, the more plentiful vortices are stronger in the ratio of the frequencies. Added to this, the stronger vortices also travel fastest because they are in a region of higher free-stream speed. The wavelengths in the wake will however (ignoring the oscillation of the period) be exactly the same as those of the weaker vortices because frequency of shedding and speed of convection are changed in the same ratio. The two sets of vortices being in relative motion, the modulation envelope no longer remains fixed relative to either set of vortices. Figure 14 shows the situation as it would develop if the vortex loops remained fixed relative to one set of vortices: before the pattern reached this stage, however, the tendency for the loops to move along to the right and so alleviate the distortion of the vortex lines would no doubt occur. In this case it appears that the modulation will travel downstream faster than the vortices. Again, the length scale in the direction of the cylinder axis will affect the process. Unfortunately, no measurements of the speed of the vortices or the modulation were made.

There are certain similarities between the waveforms we observe and the waveforms Tritton (1959) connected with his transition region. Almost certainly, what we observe is due to the variation of velocity magnitude or direction along the cylinder length and is mainly associated with the ends of the cylinder. We find no transitions to different frequency, which is the characteristic feature of Tritton's transition. It is interesting to note that Tritton found that his disturbances in the wake travelled slower than the vortex convection speed.

The photographs of Abernathy (1964) clearly show the looping-over of the vortex lines in the regions where the inclination of the vortex lines changes abruptly. The vortex-line linkages across the discontinuity, whilst not in disagreement with the ideas presented here, do not clearly corroborate them either. The cause of the discontinuous changes in Abernathy's pictures is not known.

4.4. *Reynolds number* = 235

Experiments were made at a Reynolds number of 235 using exactly the same wind-tunnel speed and hot-wire configuration as in the lower-Reynolds-number experiments. The oscillogram in figure 15 (plate 2) was obtained at 4·25 diameters behind the cylinder of diameter 0·235 cm. The bottom trace (hot-wire 2) does not show the modulation effects clearly because of overloading on the peaks. Comparison with figures 2 and 6 shows clearly that spontaneous three-dimensional effects have entered. Also there appears to be little correlation between the modulation of the waveforms from different hot-wires. The effects we now see may be peculiar to a small range of Reynolds number, for this experiment was conducted in the transition range first described by Roshko (1953). Below this range, all the vortices are composed of fluid in laminar motion, and above, all are composed of turbulent fluid. In the transition range single hot-wire investigations showed that vortices are laminar some of the time and turbulent some of the time. There is no reason to expect that the occurrence of laminar or turbulent vortices should be a two-dimensional phenomenon. The regular modulation of traces 3 and 6 in figure 15 suggest laminar-flow vortices, whereas the irregular modulation of trace 4 suggests turbulent flow.

Figures 16 and 17 (plate 3), which were obtained with a different hot-wire configuration, show some of the effects of mixed turbulent and laminar vortices more clearly. The wires were arranged in four pairs: one on each side of the wake at two downstream stations, and two positions separated along the cylinder length. In figure 16 we see that at one position (top four traces) the modulation is regular for some 100 fundamental periods, whereas at the other position the modulation is only regular in spasms and shows a mixture of low- and high-frequency irregularity. When the modulation is regular, the pairs of wires on opposite sides of the wake show that the wake has a sinusoidal centre line. Figure 17, showing only the traces from four of the wires, also shows this effect clearly.

One is tempted to draw conclusions about the vortex speed and the speed of the modulation from figures 16 and 17. Bloor & Gerrard (to be published) have shown, however, that when the vortices are turbulent, extreme care is necessary in the positioning of the hot-wires away from the centre plane of the wake before convection speeds can be measured. It must be remarked even so, that here the modulation appears to be travelling considerably slower than the vortices.

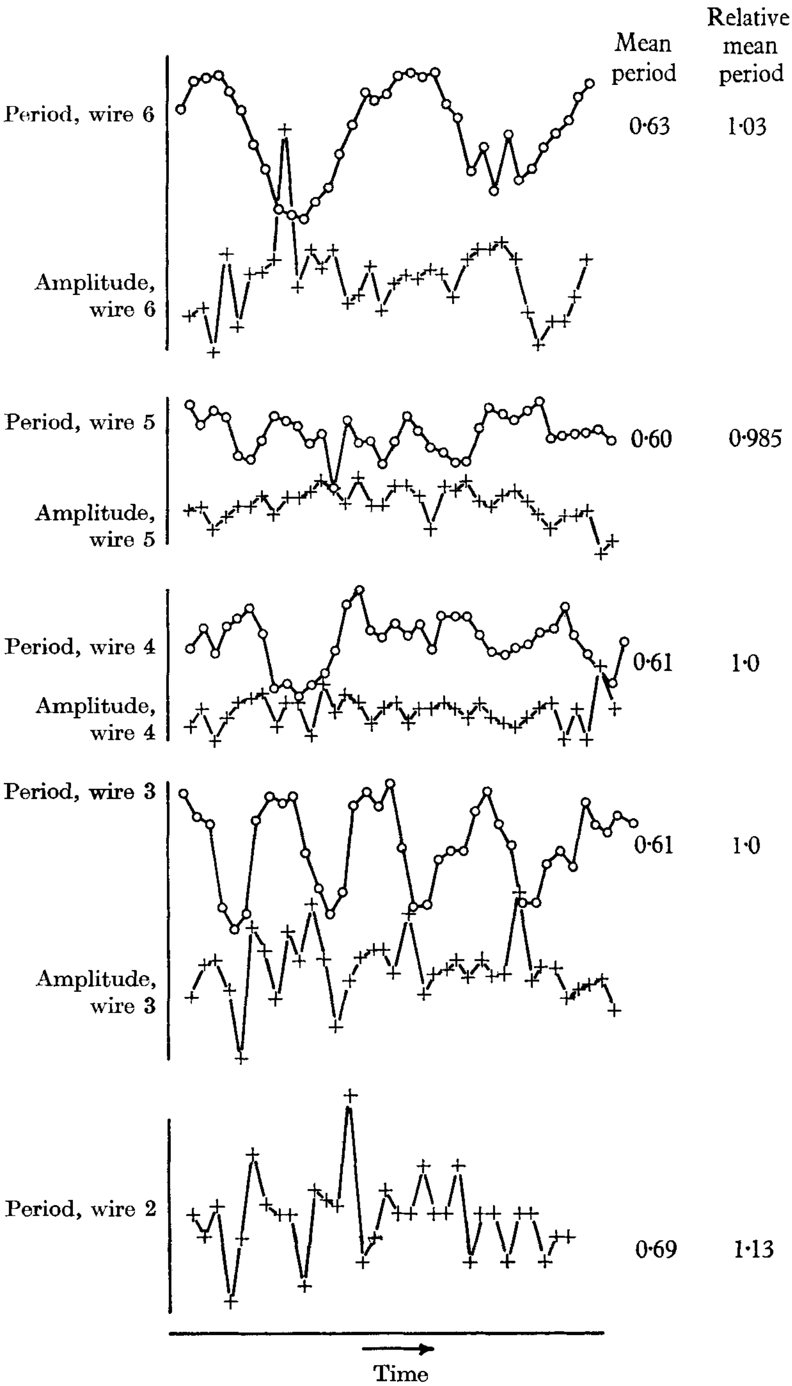

FIGURE 18. Periodic time and amplitude as a function of time, from figure 15.

Figure 18, constructed from figure 15, shows the variation of period with time contrasted with the variation of amplitude with time. Traces 3 and 6 of figure 15 show a predominantly regular modulation. The simple relationship found at low Reynolds number, of maximum modulation coupled with minimum period, is now only roughly true. It seems likely that, averaged over a long time, there would be high correlation between the positions of minimum period and maximum modulation. Lack of correlation, one presumes, is due to the frequency-determining mechanism being affected by, or even governed by, a turbulent process: governed, that is, for part of the time at these Reynolds numbers. In the case of the irregularly modulated traces, 4 and 5, we find that the period variations are irregular also. The fluctuations of period are about a mean value which is the same on all traces except number 2. This is as expected from the lower-Reynolds-number experiments.

Turbulent vortices are weaker than laminar ones. The strength of laminar vortices is about $0{\cdot}8\pi Ud$, where U is the free-stream speed and d the diameter of the cylinder, whereas turbulent vortex strengths are about $0{\cdot}5\pi Ud$ (Bloor & Gerrard, to be published). This implies the spontaneous development of vortex loops, the longitudinal elements of which separate the regions of laminar and turbulent flow. Consider two vortices of positive circulation, one laminar and one turbulent, growing together over adjacent portions of the cylinder. The laminar one will have surplus vortex strength which will loop over and join with the next laminar vortex of negative sign. If the laminar-turbulent situation still persists, the next laminar vortex of negative sign is already effectively reduced in strength by the correct amount for the remainder of its strength to balance the next turbulent vortex. This implies that the loops could all be in the same sense, that is, all going from a positive vortex to the next negative vortex nearer the cylinder. The irregular oscillations in amplitude are produced by a combination of wake oscillation and vortex-strength oscillation when turbulent vortices are present. Oscillograms taken with the hot-wires inside the wake show that often, but not always, as at low Reynolds number, the amplitude and waveform change together.

Figure 19 (plate 4) shows an oscillogram taken with the hot-wires inside the wake at 1·75 diameters downstream of the cylinder. Here we see an example of what Bloor (1964) reported, namely that the modulation of the signal is least when the hot-wire is at the end of the formation region. The waveforms are seen to be regular on the whole, with an occasional hiatus. In figure 20, the time between adjacent troughs of the waveforms (at first harmonic frequency) are plotted as a function of time. Traces 3 to 5 show that whilst the amplitude is more or less constant the period is almost constant also. The saw-tooth nature of the graphs indicates that the negative vortices follow the positive ones more closely than vice versa. We see that the hiatuses are associated with an abrupt change to the fundamental period. Such abrupt changes require further investigation, since they may well elucidate the vortex-loop-formation mechanism proposed above. It was postulated that the vortices are shed in pairs joined by a loop when the change from a laminar to a turbulent vortex occurs. If the burst of turbulent vortices ends after one of these pairs, there is a smooth transition to laminar flow. If, on the other hand, the burst ends leaving an odd member of a pair, it leaves uncompensated circulation, which is presumably passed on down the line of vortices as they grow. It is quite conceivable that a jump of the type observed may be associated with this.

A possible explanation of the increase in modulation with downstream distance from the end of the formation region may be found in the distortion

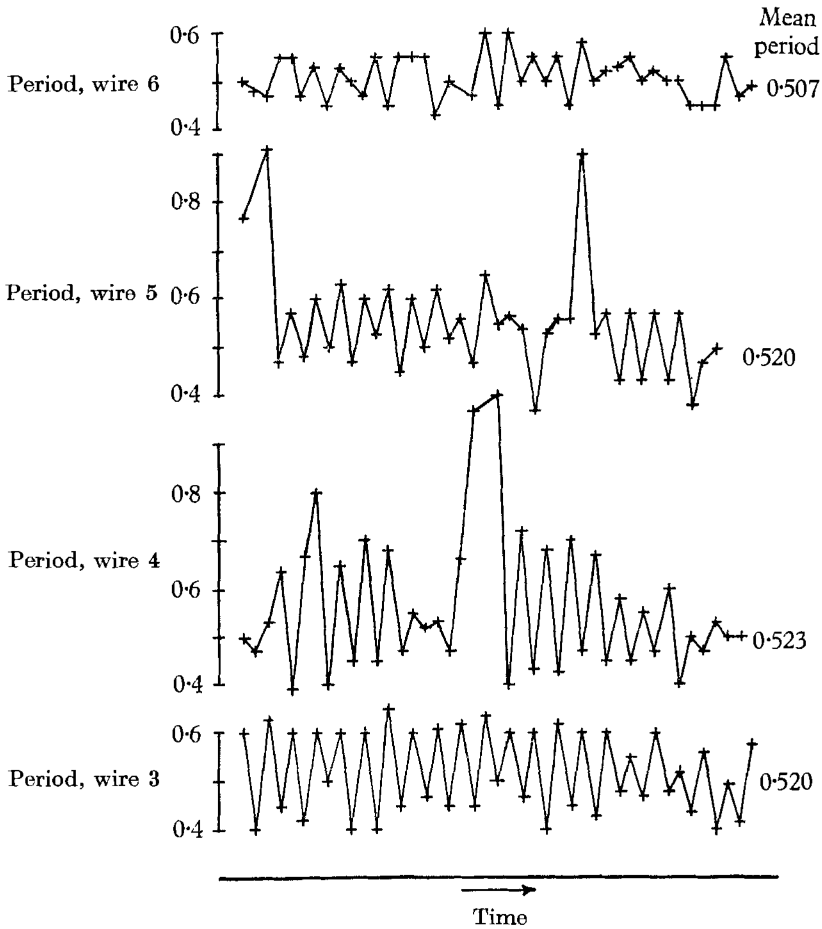

FIGURE 20. Time between adjacent troughs of the waveforms of figure 19 as a function of time.

of vortex loops. Also downstream from the formation region, the effective vortex strength diminishes as a result of interaction of vorticity shed from opposite sides of the cylinder (Bloor & Gerrard, to be published). This will also tend to increase the irregularity when the mixing is turbulent. The modulation of the hot-wire signal, seen wnen the wire is adjacent to the shear layers bounding the formation region, could be greater than that at the end of the formation region because small fluctuations in position of the shear layer produce large velocity changes at the hot-wire.

In this range of Reynolds numbers, there is much ambiguity as to the possible positions of vortex lines on plots of vortex positions constructed from the cathode-ray oscillograms. Almost certainly the vortex lines are wavy and inclined.

An idea of the downstream development can be inferred from a comparison of figure 15, obtained at 4·25 diameters downstream, and figure 21 (plate 4) at 12·3 diameters downstream. In both cases the hot-wires were outside the wake. The periods of regular modulation are more infrequent further downstream. The reason may be found in the interaction of the vortices and the distortion of the vortex loops.

4.5. *Reynolds number* $= 2 \times 10^4$

The first oscillograms obtained, actually in the process of perfecting the instrumentation, were with a 1 in. diameter cylinder spanning the wind-tunnel working section. The hot-wires were in line, parallel to the cylinder axis and separated by one diameter over the central 9 in. of the cylinder length. There is no evidence available concerning the flapping of the wake or the correlation

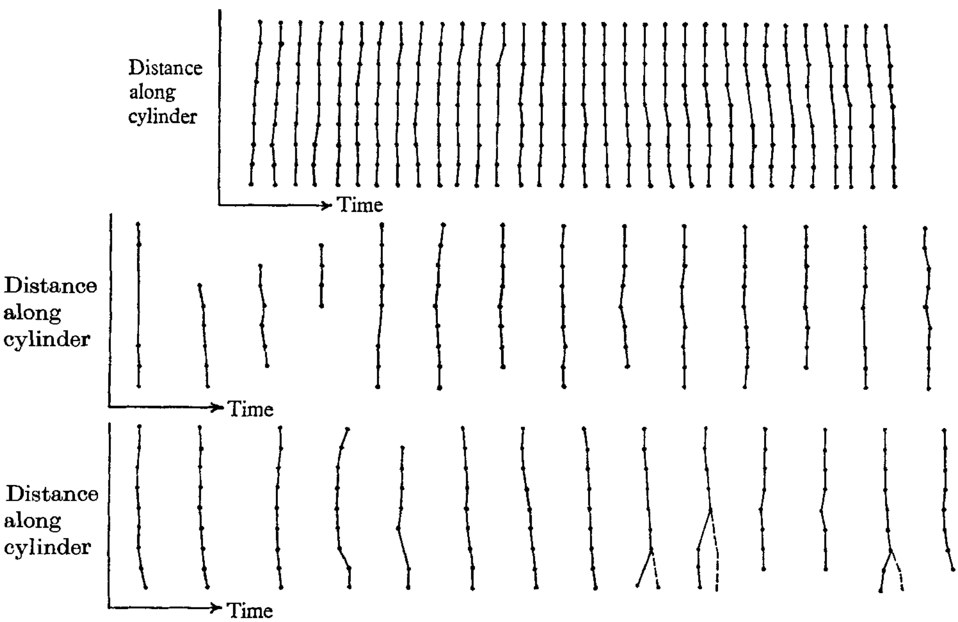

FIGURE 23. Vortex lines from figure 22 and similarly obtained oscillograms. The top figure shows every period, the lower ones every third period.

between modulation of the signal and the periodic time. When the vortices are all turbulent there is much less modulation of the signal than there is at lower Reynolds numbers, and it is more random in nature.

Measurements were only made at 3 diameters downstream and outside the wake. Figure 22 (plate 4) is an example of the oscillogram produced. From this, and oscillograms like it, figure 23 has been produced. Curves are drawn representing vortex lines on the assumption that a time history and spatial representation are similar. A brief examination with the five hot-wires used at lower

Reynolds number, in the same orientation, served to strengthen the impression gained from figure 23 that the vortex lines are nearly straight. Visualization of the flow at this Reynolds number using titanium tetrachloride also indicated the presence of vortex lines almost straight and parallel to the cylinder axis. The vortex lines appear from figure 23 to tilt backwards and forwards between the limits of inclination of $\pm 15°$. This tilting takes place at a frequency an order of magnitude less than the fundamental frequency and somewhat randomly. Kinks also seem to develop spasmodically. The gaps in figure 23 are where the waveforms were insufficiently well defined to allow measurement.

The measurement of correlation coefficient between two points separated along the length of the cylinder is complementary to an investigation of the type described here. One obtains lack of correlation because of the random differences in amplitude along the length of the cylinder.

5. Conclusions

Our understanding of the three-dimensional nature of the wake of a two-dimensional bluff body is very far from complete. All the experimental work previously performed shows a variety of forms of three-dimensional characteristics. Some of this variety may be attributed to the differences in free-stream conditions. Of the present experiments those at $R = 85$ are thought to be typical of all Reynolds numbers at which vortices are shed below Tritton's transition at about $R = 90$. The measurements at $R = 235$ are expected to typify the transition range of Reynolds number in which the vortices change from being composed of fluid in laminar motion to turbulent vortices. The measurements at high Reynolds number may be typical of all wakes of two-dimensional bluff bodies with turbulent vortices, but our findings in this range contribute very little. As pointed out in the introduction, there are other ranges of Reynolds number at which three-dimensional effects arise from special causes.

At low Reynolds number, the main experimental findings are that the modulation of the hot-wire signal is directly related to the difference in vortex-shedding frequency and that the modulation is the result of the flapping of the wake in the manner of a flag. The same effects are present at higher Reynolds number. In the transition range ($R = 235$) three-dimensional effects result from the occurrence of laminar and turbulent vortices at the same time behind the cylinder at different positions along its length. The higher Reynolds number results are also complicated by the effects of turbulent mixing and consequent fluctuations of vortex strength. The results can be explained in part on the basis of the suggested formation of vortex loops.

Points, where further investigation along the same lines as those described here would seem profitable, have been indicated. It seems reasonable to concentrate first on the stable range of Reynolds number where the effects of simple changes can be studied without the complications attending the presence of turbulent flow. We will list now the further investigations which seem necessary and the questions which arise from the present work.

Why are straight vortex lines inclined? Is this a reflexion of free-stream non-uniformity or not? Does the inclination change with time at low Reynolds number?

The speed of convection of the modulations needs to be measured. Are the short and long period modulations similar? Why is the modulation correlated with the variation of period with time? It is perhaps difficult to attempt to answer this question before an explanation of what indeed determines the period itself is given.

Where turbulent vortices are present, observations at the end of the formation region were seen to merit further investigation. The downstream development in the presence of turbulent vortices requires further study.

Simple towing experiments in still water may serve to elucidate the vortex-loop mechanism. A cylinder with a step change and a gradual change in diameter at some point along its length produces a simple configuration. This is almost a repeat of Taneda's (1952) experiment. The further investigation of the wake of a tapered body may well be the next stage in such an experiment. It would be interesting to see the effect of towing a circular cylinder in a stratified fluid where the Reynolds number varied along the length. The production of turbulent vortices over a section of the length by the injection of a disturbance in the boundary layer may be possible.

Obviously there is a wealth of experimental and theoretical work waiting to be done in this field. It is the author's hope that further investigations along the lines indicated may prove a fruitful line of attack on this problem.

The experimental work was done at the Mechanics of Fluids Laboratory at Manchester University at the end of 1964. The reduction of results and formulation of ideas took place whilst the author was on leave of absence at the Pennsylvania State University, Department of Aeronautical Engineering, during 1965. I would like to thank the members of both departments for their co-operation in making this leave possible.

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